DOCUMENT RESUME

ED 437 162		PS 027 721

AUTHOR	Hunting, Robert P.
TITLE	Rational Number Learning in the Early Years: What is Possible?
PUB DATE	1999-00-00
NOTE	22p.
PUB TYPE	Reports - Research (143)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	Cognitive Development; Computation; *Concept Formation; Division; Elementary School Students; Fractions; Number Concepts; *Numeracy; Primary Education; *Sharing Behavior
IDENTIFIERS	Distributive Counting

ABSTRACT

This report describes an investigation of how young children respond to two types of tasks: (1) finding one-half of a continuous and a discrete material; and (2) attempting to share continuous and discrete material equally between two dolls. Continuous material, such as string, paper, or liquid, is quantified by adults using measurement units. A question often asked about this kind of material is "How much?" Discrete material, such as buttons or beads, is quantified by counting--asking "How many?" The investigation was interested in both the accuracy of the children's responses and the methods they used. The performance of one child, Julie, classified as a weak "halver," was examined in detail to explore how social practices are bound up with rational number knowledge, including serial sharing and parallel sharing. Based on findings, the report asserts that if people want young children to learn mathematics in a meaningful way, it is important to identify situations in which quantities can be shared using methods familiar to children. Sources of children's knowledge, experience, and motivation to subdivide quantities include the primitive personality of the child, the modeling behavior of parents and other adults, and interactions with siblings and peers. The report further explores children's abilities in subdividing quantities by discussing distributive counting, the process by which children systematically allocate items resulting in equal shares. The report discusses the different ways in which children "deal," or the strategies they use to equalize shares, then describes the unique role the fraction one-half plays in the development of children's rational number knowledge. A second study examining strategies used by preschoolers when counting and sharing is also noted. The report concludes that teachers should be able to involve young children in problems of sharing even if those children have not yet become rational counters, and that sharing tasks may assist children's developing counting skills through opportunities to check or verify the sizes of shares. (Contains 34 references.) (EV)

Rational Number Learning in the Early Years: What is Possible?

Robert P. Hunting

Professor of Mathematics Education

East Carolina University

Mathematics Dept

230 Austin Building

Greenville, NC 27858

Ph. 919 328 1896

Fax: 919 328 6414

Email: huntingr@mail.ecu.edu

Rational Number Learning in the Early Years: What is Possible?

The Mathematics of the Young Child

Rational Number Learning in the Preschool Years: What is Possible?

The *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989) recognizes that "Children enter kindergarten with considerable mathematical experience, a partial understanding of many concepts, and some important skills, including counting" (p. 16). Considerable research has accumulated to support this statement (Gelman & Gallistel, 1978; Hughes, 1986; Irwin, 1995; Miller, 1984; Kamii, 1985; Resnick, 1989; Wright, 1991; Young-Loveridge, 1987).

Young children have a variety of meanings and responses to tasks involving the subdivision of physical quantities, such as food in various forms, and other material. It is important to understand children's conceptions because subdividing or breaking up material is a critical source of experience for children. Historically, the mathematics of fractions is based on the notion of breaking up or "fracturing" quantities. In the preschool years children's experiences of subdivisions of quantities, and their methods for creating equal shares, form what is called *pre-fraction knowledge*. Pre-fraction knowledge includes the social and cognitive foundations of experience that underpins the formal notation we teach our children as part of their cultural heritage. Symbols such as numerals, words, and even graphic images have no meaning in and of themselves. Meaningful mathematics learning occurs when each child associates some personal experience—grounded in action, or negotiated through social interactions with others—with symbols. The challenge for teachers of young children is to determine possible meanings that might be available to children, or which could reasonably be expected to be made available through the social interactions of home and school. Available meanings can then serve as platforms upon which the formal symbolism of mathematics can be associated.

The Case of Julie

Julie, aged 5 years 3 months, participated in an investigation of how young children respond to two types of tasks: 1) finding one half of given continuous and discrete material, and 2) attempting to share continuous and discrete material equally between two dolls (Hunting & Davis, 1991). Continuous material, such as string, paper, or liquid, is quantified by adults using measurement units. A question often asked about this kind of material is “how much?” Discrete material, such as buttons or beads, is quantified by counting. “How many” is a question asked of discrete material. For this investigation, we were interested in both the accuracy of the children’s responses and the methods they used. Julie as well as 74 other children completed the study. The average age of the children was 4 years, 11 months.

Task One: Finding One Half

One-half is a fundamental building block in elementary mathematics. We wanted to know how young children thought about this number. Specifically, we were interested in how children’s ideas about the fraction one half develop from a qualitative to a quantitative conception. Compared to the other children in the study, Julie was classified as a weak "halver". In the continuous quantity half problem, involving a candy stick made of soft but rigid candy approximately 110 mm long, Julie was told by the interviewer: “I would like half of this candy stick. Can you give half to me?” Julie cut a 30 mm long piece as half of the candystick for the interviewer, leaving 80 mm remaining. For the discrete problem half of 12 jelly babies (for Julie) was eight. The interviewer had placed 12 jelly babies of uniform color on a saucer and said: “We need to save half of them for a friend. Can you help me put half of those jelly babies on this (another) saucer?” Julie first placed three jelly babies aside one at a time, then picked up two together, finally placing aside one handful of three.

Task Two: Sharing

Julie's performance on the two sharing tasks was much more impressive. For the continuous quantity sharing task she was asked "Can you give the dolls the candy snake so that each doll gets an even share?" Initially Julie attempted to share the candy snake, approximately 150 mm long, between two dolls, by tearing it apart manually. When this strategy failed she used a knife. Julie cut two small pieces of length 20 and 18 mm for each doll, then stopped. She was encouraged to share up all of the candy. Julie continued cutting small pieces and allocating them in order until there was one piece left. The interviewer again encouraged Julie to consider how to deal with the remainder, which was duly divided once more. Pieces given to the first doll measured 20, 19, 16, 13, 9, and 5 mm (total 82 mm); pieces given to the second doll measured 18, 19, 9, 15, 10, and 9 mm (total 80 mm).

For the discrete sharing task 12 cracker biscuits were placed on the table, in a single pile, near the two dolls. The child was told, "Mother wants all the crackers to be shared evenly. Can you share all the crackers so that each doll has the same?" If the child stopped before all crackers were allocated, she was asked if all the crackers had been given out, and encouraged to continue. On completion of the task the child was asked if each doll had an even share, and to tell how she knew. Julie used a systematic one-to-one dealing strategy until there were two of the 12 crackers left. At this point she started to place the eleventh cracker in front of the doll that had received the 10th cracker, but changed her mind and placed it in front of the first doll. She then stopped. The interviewer encouraged Julie to allocate the remaining cracker, which she did by also placing it out for the first doll so that this doll had seven crackers. Julie agreed the dolls would be satisfied.

Julie's Performance

Julie's performance on the "half" tasks placed her in the lower one-third of the 75 children interviewed. She subdivided the candystick once, resulting in portions measuring 30 and 80 mm (45% error). Eight of the 12 jelly babies were saved as half. Yet Julie had a dealing procedure which allowed her to create equal quantities. She used this method with the continuous material in the Snakes task. Also in the Crackers problem this method worked until she seemed to lose track of which doll was to receive the 11th cracker. She did not use counting or measuring strategies to verify or check her efforts. Julie showed that she had the necessary cognitive skill to subdivide small collections into equal amounts. Her progress toward a more quantitative understanding of one-half would be enhanced if her dealing procedure could become the action-meaning base for the language and symbolism for this number.

Social Practices

Rational number knowledge in its genesis is inextricably bound up with the social practices and politics of regulating the use of a commodity—very often food. Ancient cultures, such as the Egyptians, used fractions to calculate the areas of fields, for example. Teachers need to be aware of subtle nuances involved in social interactions. Sharing of a commodity can be done in two main ways. *Serial* sharing occurs when one item is distributed between two or more people over time. *Parallel* sharing requires the subdivision of a quantity enabling simultaneous use by two or more. An important common feature of both serial and parallel approaches, in many cases, is to achieve a goal of approximating equal access to the commodity. Much sharing is serial rather than parallel because, as one parent explained, "we only have one of a lot of toys". Serial sharing would also be expected of larger play-things such as swings, bikes, and trampolines.

Interactions with a sibling may be important for learning how to gain a fair share. The roles of the younger and the elder sibling in regulating their respective desires for as much material as they each can get is a complex issue to investigate. Younger children are more likely to trust others to decide how much food they will receive, but trust may be "unlearned" progressively due to personality factors as well as experience. Parents seem to encourage or at least accept that older children can have more—so long as the younger child doesn't complain! In a study of the social origins of sharing in 3 year olds (Hunting, 1991) parents were interviewed to discover possible explanations for the cognitive skill of sharing. One couple related how cookies were given to their children while travelling in the car. The three year old child would eat two cookies in the time the younger sister (not yet two years old) would take to eat one. The younger sister would not complain when her brother would ask for another cookie so long as she had some cookie left to eat. On the other hand, another parent proposed a scenario where an even number of candies is given for two siblings to share. The younger child, being naturally greedy, takes a larger amount. The elder then insists on a one-to-one deal-out in order to guarantee at least an equal quantity for himself. As one parent argued, sharing is a survival skill. What we do not properly understand is the extent to which parents' decisions are determined by the "squeakiest wheel", and hence depend on personality and dispositional traits. Alternatively, parental influence and intervention may depend on their personal tolerance to the demands of one or more of their offspring. Which of two siblings is likely to be more persistent? Is there a certain age when the younger child begins to realize an older sibling is (1) receiving more than he is, and (2) realizes that such a state of affairs is something that is within their power to change? A fundamental determinant in the politics of fair shares would seem to be discrimination of which of two quantities is larger.

Linked to young children's understanding of differences between amounts is the intervention of the parent in sharing situations. Appearing in transcripts of mother/child conversations (Tizard and Hughes, 1984), Walkerdine's (1988) study of how relational terms such as more and less are used indicated that almost all the examples involved regulation of consumption:

> In every case initiated by the child, she either wants more precious commodities, of which the mother sees it as her duty to limit consumption, or the child does not want to finish food which the mother sees it her duty to make the child eat (p. 26).

Since our culture is concerned about the regulation of consumption of commodities, the term *more* is better understood; the term *less* not as well. A conception of "equalness" would seem to depend on the existence, for the child, of an equilibrium between the two complementary situations of more and less. If one gets more, then the other gets less, assuming the commodity is finite at the time. Greater stress on *more* may inhibit the development of notions of equal quantities. Complex relationships then would appear to exist between the dynamics of siblings, parent and child, and apprehension of equality, including limiting of a desire to have more.

Parents have reported observing personality differences between their offspring which they cannot account for in terms of child-rearing practices. How personality traits interact in the negotiations related to sharing of commodities in the early years is not well understood. An easy-going placid child may not feel motivated to attend to distinctions between different shares of commodities that might be highly engaging to another child. Also, children who have not had experience in participating in decisions about how commodities will be distributed may lack awareness of quantitative differences between shares. Practices of sharing and distribution need to be observed at home over extended periods. It is possible that a practice common across cultures is the apportionment of food to individuals from a common reservoir. Such practice is likely to be highly salient to a hungry child, and as such, noticed well! Perhaps it is here that a dealing procedure begins to take root.

Teachers should be aware that three year olds have many more opportunities to observe food and other material being shared than opportunities to personally participate in its subdivision. It is likely that the role of observation, and the significance of what it is that is *noticed*, may be critical in children's acquisition of dealing knowledge. Teachers may find it worthwhile to set up home-like play situations involving sharing, and take the opportunity to talk with the children about what they think is involved and why they take certain actions. In Figure 1, Nick (age 3 years 6 months) and Joanne (age 3 years 7 months) are invited to cook a meal for two dolls. After setting the table, and preparing the "food" (white plastic counters) on a toy stove (in the background), they serve it. Nick places 5 items on each plate. The teacher asks the children if the dolls get the same amount of food, and if they know a way to check. Joanne counts out loud the items on the plate on the right: "one, two, three, four, five". The teacher asks Nick to check if the doll nearest him gets the same. Nick counts: "one, three, four, eight, six". While Nick is yet to count in an orthodox way, he was able to subdivide the items equally.

Figure 1 about here

If we want young children to learn mathematics in a meaningful way, then it will be important to identify situations in which quantities can be shared, using methods familiar to the children. While we know that a significant number of young children learn how to subdivide quantities using systematic methods, how such methods are learned is not known exactly. We can point to several potentially important sources of knowledge, experience, and motivation.

First is the primitive personality of the child. By primitive we mean that which is genetically given, as well as environmental influences that are at work during the earliest months of life. A child's early feelings of security may dictate how placid or competitive that child will be in social situations. Second, there is the modeling behavior of parents and other significant adults, as they regulate food and other salient commodities, and ways they respond to various ploys of their offspring to get what they want. Third, there are interactions with siblings and peers. It is reasonable to assume that not all young children "care" uniformly when it comes to determining equal quantities. Fairness is located in the mind of the individual. What one child considers fair may be quite unfair for another child. How a social or group consensus develops about fairness and equality, may involve negotiations between the individuals involved over periods of time. How one member views the outcome of a sharing situation may be influenced by the views of other, more dominant members. Fourth, the commodity itself may be able to be shared only in certain ways, as for example, the case of children sharing one plaything.

Children's Abilities Subdividing Quantities

Recall that Julie was given sharing tasks requiring continuous and discrete materials to be shared between two or three dolls. The continuous material was a candy snake; the discrete material was 12 crackers. Each task seemed to require different kinds of solution strategies. The snakes task involves subdividing a continuous quantity. The crackers task involves subdividing a discontinuous or discrete quantity. Our research as well as others (Hiebert & Tonnessen, 1978; Miller, 1984) suggests that children use different kinds of mental processes to solve each type of task. One kind of process is the *dealing procedure* (Davis & Pitkethly, 1990), such as when a deck of playing cards is distributed equally between the players. This approach is used successfully for tasks involving items such as crackers. The second kind of process involves intuitive estimation and measurement strategies, where eyeballing end-points and estimating the place where the first cut will be made is of importance. Here a more holistic global appraisal seems to take place.

The process by which children systematically allocate items resulting in equal shares is also known as *distributive counting* (Miller, 1984), or *splitting* (Confrey, 1995). A significant number of pre-schoolers possess a powerful, systematic dealing method. In one study (Hunting & Sharpley, 1988) 60 per cent of a sample of 206 four and five year olds were observed to successfully allocate 12 items to three dolls using a dealing procedure. In the study in which Julie participated, 58 (77 per cent) of 75 four and five year olds (Hunting & Davis, 1991) were able to share 12 cracker biscuits between 2 dolls using systematic methods. While children who use systematic methods do not necessarily know at the end of it how many items each doll has received, the method guarantees that each doll receives an equal number of items. Such a method is an ideal action-meaning base for the mathematical language and symbolism used to represent numbers we know as fractions, particularly unit fractions such as $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. Sophisticated counting knowledge is not required to produce the required shares, although counting can be used to verify solutions.

Young children "deal" in different ways

There are variations in the ways young children successfully distribute a discrete quantity:

- *one to one* (one item is given to a recipient at a time, in turn). Children who use systematic methods can commence each cycle from the same position, as for example, Sharlene (see Figure 2), or from different positions, as in the example of Jim (see Figure 3). In the figures below, A, B and C represent dolls and the dots represent crackers. The flow of action proceeds from left to right.

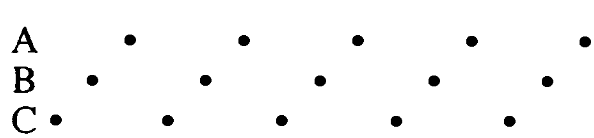

Figure 2: Sharlene's cyclic and regular solution strategy, always beginning with Doll C

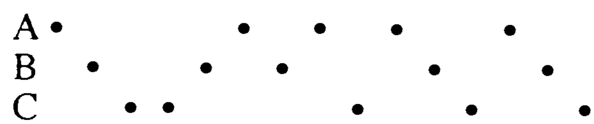

Figure 3: Jim's cyclic and irregular solution strategy, beginning with a different doll each cycle

- *many to one* (two or three items are given to each recipient, in turn);
- *combinations of the above* (it is common to see a child begin allocating items in lots of two or three for a cycle or more, then continue using a one to one action); and
- *non-systematically* or by trial and error. For example, Carla placed out 5 items, one item at a time, in front of the first doll, 7 items, one item at a time, in front of the second doll, and the remaining items in front of the last doll (see Figure 4), again, one at a time.

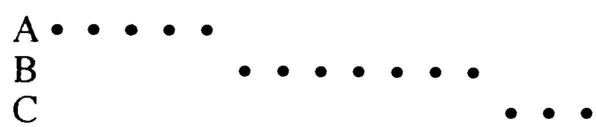

Figure 4: Carla's solution strategy

In contrast, Tess appeared to use a systematic method, since each item was allocated to the "next" or adjacent doll. However regularity in the order of allocation was not maintained, resulting in unequal shares (see Figure 5).

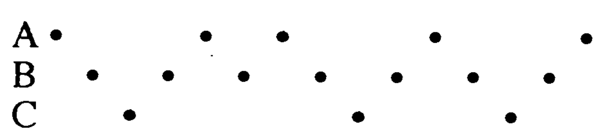

Figure 5: Tess's solution strategy

Teachers should observe carefully how young children go about apportioning items. In particular, the order in which each cycle commences should be noted. For example, a child might be observed to commence the dealing cycle from a different doll each time, as in the example of five year old Jim (Hunting, Pepper, & Gibson, 1992). Variation of starting position indicates an advance in thinking, and a readiness to progress. Variation of starting position is evidence that the child is monitoring an internal cycle of "lots"—in this case, threes, rather than physical perceptual markers such as dolls

Other strategies used to equalize shares

In addition to systematic dealing, young children have been observed to use a variety of other methods to equalize shares including

- *subitizing*. This technical term (Kaufman, Lord, Reese, & Volkmann, 1949) means the ability to apprehend numerosities of small collections—up to five items—without counting;
- *informal length or height comparisons*. We have observed children place items in columns in front of the recipient dolls, making it possible to observe unequal lengths which can be equalized. Stacking items with uniform thickness (such as cracker biscuits) in front of the dolls allows a child to use visual height estimates as evidenced by eye movements from stack to stack; and
- *counting*. Children who are rational counters will check the results of their sharing actions by counting the numbers of items for each recipient.

The Fraction One-Half

The number one-half plays a unique role in the development of children's rational number knowledge. It is the fractional number that almost all children learn first, and it is a fraction that many use fluently. Research studies (Bergeron & Herscovics, 1987; Campbell, 1975; Clement, 1980; Clements & Del Campo, 1987; Kieren & Nelson, 1978; Piaget, Inhelder, & Szeminska, 1960; Polkinghorne, 1935; Pothier & Sawada, 1983;

Sebold, 1946; Watanabe, 1996) indicate that the fraction one-half is well supported by the operation of subdividing a given continuous quantity into two portions, and appears to become established at an early age compared to knowledge of other fractions.

Julie could be said to have a qualitative understanding of the fraction one half. She made just one rather unequal subdivision of the candy stick, and divided 12 jelly babies into subsets of 8 and 4. Knowledge of one-half develops from that of a qualitative unit to a quantitative unit. In the study of 75 four and five year olds described earlier, more than 50 per cent of these children were accurate to within 10 per cent in halving a continuous quantity. Most children performed just one subdivision (82 per cent). Multiple subdivisions were of two types: "algorithmic halving" (Pothier & Sawada, 1983), or a series of subdivisions in a linear sequence along the length of the material. Jelly babies were used to find one-half of 12 discrete items. Forty per cent of these children placed exactly 6 jelly babies aside. A further 34 per cent chose either 5 or 7 jelly babies. Two main procedures were used: thirty six children placed the jelly babies aside one at a time; 37 children placed out handfuls. Of the children who used a one-at-a-time method 10 audibly counted out as they proceeded, six out of 11 made adjustments after checking their outcomes, while 23 did not check at all. For the children who used handfuls, six checked their result by counting, 10 made adjustments after the interviewer asked if half the jelly babies had been put aside, while 26 children did not check overtly.

For the sharing tasks 73 per cent of the children were accurate to within 10 per cent of the mid point of the continuous candy snake. Single subdivision was the most common method (77 per cent). Fifty one of 58 children made no effort to check their result. Four children made adjustments prior to subdividing; 3 children checked and adjusted after subdivision. Multiple subdivisions were made by 5 children, all of whom used the linear sequence method. In the discrete sharing task, involving cracker biscuits, 89 per cent allocated 6 crackers to each doll; 77 per cent using a systematic dealing procedure.

Establishing what one half means for another is a critical task, especially for the teacher of preschool and elementary children. How to stimulate and deepen the child's conception of one half is at the core of effective mathematics education. It is worth noting that even though children's early experiences of fraction terminology are located in continuous quantity problems and events, their facility at creating precise units with discrete material using a reliable and generalizable dealing procedure should not be ignored or under-estimated. Instruction should be designed to extend children's meaning base for one-half to problems involving discrete items, where systematic dealing procedures can be applied. In this way one-half as a mathematical object can develop from a qualitative unit to a quantitative unit in children's thinking.

Counting and Sharing

The cognitive skills of both counting and sharing develop during the early childhood years. Both skills require action on discrete elements, including the logic of one-to-one correspondence. Two studies (Pepper, 1991; Pepper & Hunting, 1998) examined the relationship between young children's sharing and counting abilities. In the first study 75 four and five year old children were given sharing and counting tasks in separate interviews (Pepper, 1991). Children were classified into poor, developing, and good counters based on Steffe's theory of counting types (Steffe, von Glasersfeld, Richards, & Cobb, 1983). Dealing competence was found not to relate directly to counting competence for problems requiring the distribution of discrete items from a single group of items. Children demonstrated the use of systematic dealing procedures, where no obvious use of counting or measurement skills were observed. Children classified as poor counters were able to equally divide groups of discrete items and be confident about the result.

A second study (Pepper & Hunting, 1998) was conducted to examine strategies 25 preschool children used when counting and sharing. Visual cues such as subitizing and informal measurement skills were restricted. A task was devised in which coins were to be distributed equally into money boxes. After the coins were deposited into the money boxes the children were no longer able to manipulate the coins or visually check the size of shares. Children who were successful on this task may have mentally marked a particular box as a signpost to indicate where a new cycle would commence. Another feature of some children's distributions was an "adjacent box" strategy, whereby they would deposit the next coin into the box adjacent to the box into which the previous coin had been deposited. Before a coin was deposited, the child would pause as if replaying actions already performed during the cycle. Often the correct decision was made as to where that coin should be deposited. It is possible that these children were keeping a mental record of their actions. Results from the second study supported the view that an ability to equally distribute groups of items does not depend on advanced counting skills.

What this research means is that teachers should be able to involve young children in problems of sharing even if those children have not yet become rational counters. Indeed, sharing tasks may assist children's developing counting skills through opportunities to check or verify the sizes of shares. The type of task given to children will determine the extent to which counting may be needed. Items numbering less than 12 may be shared successfully without counting. Tasks involving larger numbers would encourage counting. Tasks that prevent children from scanning relative sizes of shares could be presented by having items placed in containers where perceptual cues are eliminated. For problems with small numbers, such as eight coins to be distributed equally into two opaque boxes, children could be allowed to see the coins in one box but only be told the number of coins in the second box. Children who need perceptually accessible material would be encouraged to point or make verbal counts for the hidden items in order to find the total number of coins. In this case, a child may realize that saying "four" can stand for four items that no longer need to be counted. Such a task could be extended to include problems with larger numbers.

Final Comments

McLennan and Dewey (1895) in their classic work, *The Psychology of Number*, argued that "the psychical process by which number is formed is from first to last essentially a process of 'fractioning'- making a whole into equal parts and remaking the whole from the parts" (p. 138). Much effort is expended in primary elementary classrooms teaching children whole number concepts. A fundamental activity involves enumerating and ordering collections of items, where the focus of attention is on the individual item as a "one", whole, or unit. The numerical attribute of the collection as a unit can be determined by counting, or using addition strategies. Children will impose their own structure on a collection by segmenting it (perhaps not physically) into sub-units, in order to accomplish their goals. For example, a collection of 15 items (whose numerosity is known to the teacher but not the child) might be segmented into three units each of 5 items, because the number sequence of fives is familiar. All it takes is a change of focus, provoked by an appropriate problem context, to reconsider a collection of 15 items as a "one", and restructure it as three units of five. Numerical relationships between various units of different magnitudes require vocabulary and symbols to precisely and unambiguously describe those relationships. Sub-dividing collections based around sharing problems can provide opportunities for introducing basic vocabulary of rational numbers. It is also the case that opportunities to sub-divide collections can be used to introduce whole number division, including meaning for the ÷ symbol. There is research evidence that significant numbers of preschool children can subdivide quantities into equal sub-units. Fraction vocabulary can be introduced naturally in the discussions that ensue between children and teacher. The fact that the dealing strategy will work, in a general sense, no matter how many recipients are involved, suggests that vocabulary for unit fractions such as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and so on, might be introduced in preschool years, even if symbols are not.

References

Bergeron, Jacques C., & Herscovics, Nicholas. Unit Fractions of a Continuous Whole. In Jacques C. Bergeron, Nicholas Herscovics, & Carolyn Kieran (Eds.), *Proceedings of the Eleventh International Conference of Psychology of Mathematical Education* Vol 1, pp. 357 - 365. Monitorial, 1987.

Campbell, Bobby G. The Child's Understanding of Three Interpretations of Certain Unit Fractions Prior to Formal Instruction (Doctoral dissertation, The University of Oklahoma, 1974). *Dissertation Abstracts International, 35*, 4855A, 1975.

Clement, John Cognitive Microanalysis: An Approach to Analyzing Intuitive Mathematical Reasoning Processes. In Sigrid Wagner & William E. Geeslin (Eds.), *Modeling Mathematical Cognitive Development*. Columbus, OH: ERIC Clearinghouse for Science, Mathematics and Environmental Education, 1980.

Clements, McKenzie A., & Del Campo, Gina *Fractional Understandings of Fractions: Variations in Children's Understanding of Fractional Concepts, Across Embodiments (Grades 2 through 5)*. Paper presented at the Second International Seminar on Misconceptions and Educational Strategies in Science and Mathematics, Cornell University, Ithaca NY, April 1987.

Confrey, Jere, Student Voice in Examining "Splitting" as an Approach to Ratio, Proportions and Fractions. In Luciano Meira & David Carraher (Eds.), *Proceedings of the 19th Annual Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 3-29). Recife: Universidade Federal de Pernambuco, 1995.

Davis, Gary E., & Pitkethly, Anne. Cognitive Aspects of Sharing. *Journal for Research in Mathematics Education, 21*, 145-153, 1990.

Gelman, Rochel, & Gallistel, C. R. *The child's understanding of number.* Cambridge, MA: Harvard University Press, 1978.

Hiebert, James & Tonnessen, Lowell. H. Development of the Fraction Concept in Two Physical Contexts: An Exploratory Investigation. *Journal for Research in Mathematics Education, 9*, 374-378, 1978.

Hughes, Martin *Children and Number; Difficulties in Learning Mathematics.* Oxford: Blackwell, 1986.

Hunting, Robert P. The Social Origins of Pre-Fraction Knowledge in Three Year Olds. In Robert P. Hunting & Gary Davis (Eds.), *Early Fraction Learning* (pp. 55-72). New York: Springer-Verlag, 1991.

Hunting, Robert P., & Davis, Gary. Dimensions of Young Children's Conceptions of the Fraction One Half. In Robert P. Hunting & Gary Davis (Eds.), *Early Fraction Learning* (pp. 27-53). New York: Springer-Verlag, 1991.

Hunting, Robert P., Pepper, Kristine L., & Gibson, Sandra J. Preschoolers' Schemes for Solving Partitioning Tasks. In William Geeslin & K. Graham (Eds.), *Proceedings of the Sixteenth PME Conference (Vol. 1)* (pp. 281-288). Durham NH: The University of New Hampshire, 1992.

Hunting, Robert P., & Sharpley, Christopher F. Preschoolers' Cognitions of Fractional Units. *British Journal of Educational Psychology, 58*, 172-183, 1988.

Irwin, Kathryn. C. Children's Understanding of the Principles of Covariation and Compensation in Part-Whole Relationships. *Journal for Research in Mathematics Education, 27*, 25-40, 1995.

Kamii, Constance *Young Children Reinvent Arithmetic: Implications of Piaget's Theory*. New York: Teachers College, 1985.

Kaufman, E. L., Lord, M. W., Reese, T. W., & Volkmann, J. The Discrimination of Visual Number. *American Journal of Psychology, 62*, 498-525, 1949.

Kieren, Thomas E., & Nelson, Doyal. The Operator Construct of Rational Numbers in Childhood and Adolescence - An Exploratory Study. *The Alberta Journal of Educational Research, 24*(1), 1978.

McLennan, James A., & Dewey, John. *The Psychology of Number*. New York: D. Appleton & Co., 1895.

Miller, Kevin. Child as the Measurer of All Things: Measurement Procedures and the Development of Quantitative Concepts. In Catherine Sophian (Ed.), *Origins of Cognitive Skills: The Eighteenth Annual Carnegie Symposium on Cognition* (pp. 193-228). Hillsdale, NJ: Erlbaum, 1984.

National Council of Teachers of Mathematics. *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author, 1989.

Pepper, Kristine. L. Preschoolers' Knowledge of Counting and Sharing in Discrete Quantity Settings. In Robert P. Hunting & Gary Davis (Eds.), *Early Fraction Learning* (pp. 103-129). New York: Springer-Verlag, 1991.

Pepper, Kristine L., & Hunting, Robert P. Preschoolers' Counting and Sharing. *Journal for Research in Mathematics Education.*, *29*, 164-183, 1998.

Piaget, Jean, Inhelder, Barbel, & Szeminska, Anna. *The Child's Conception of Geometry*. New York: Basic Books, 1960.

Pitkethly, Anne, & Hunting, Robert P. A Review of Recent Research on Initial Fraction Learning. *Educational Studies in Mathematics, 30,* 5-38, 1996

Polkinghorne, Ada R. Young Children and Fractions. *Childhood Education, 11*, 354-358, 1935.

Pothier, Yvonne, & Sawada, Daiyo. Partitioning: The Emergence of Rational Number Ideas in Young Children. *Journal for Research in Mathematics Education, 14*, 307-317, 1983.

Resnick, Lauren B. Developing Mathematical Knowledge. *American Psychologist, 44*, 162-169, 1989.

Sebold, Mary T. Learning the Basic Concepts of Fractions and their Application in the Addition and Subtraction of Simple Fractions. *The Catholic University of America Educational Research Monographs, 14*(2), 1-88, 1946

Steffe, Leslie P., von Glasersfeld, Ernst, Richards, John, & Cobb, Paul. *Children's Counting Types: Philosophy, Theory, and Application*. New York: Praegar Scientific, 1983.

Tizard, Barbara, & Hughes, Martin. *Young Children Learning*. London: Fontana, 1984.

Walkerdine, Valerie. *The Mastery of Reason: Cognitive Development and the Production of Rationality*. London: Routledge, 1988.

Watanabe, Tad. Ben's Understanding of One-Half. *Teaching Children Mathematics, 2*, 460-464, 1996.

Wright, Robert J. What Number Knowledge is Possessed by Children Entering the Kindergarten Year of School? *Mathematics Education Research Journal, 3*(1), 1-16, 1991.

Young-Loveridge, Jennifer M. Learning Mathematics. *British Journal of Developmental Psychology, 5*, 155-167, 1987.

U.S. Department of Education
Office of Educational Research and Improvement (OERI)
National Library of Education (NLE)
Educational Resources Information Center (ERIC)

ERIC®

REPRODUCTION RELEASE
(Specific Document)

I. DOCUMENT IDENTIFICATION:

Title: RATIONAL NUMBER LEARNING IN THE EARLY YEARS: WHAT IS POSSIBLE?	
Author(s): ROBERT P. HUNTING	
Corporate Source: COPLEY, J.V. (Ed.) MATHEMATICS IN THE EARLY YEARS, NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS	Publication Date: 1999

II. REPRODUCTION RELEASE:

In order to disseminate as widely as possible timely and significant materials of interest to the educational community, documents announced in the monthly abstract journal of the ERIC system, *Resources in Education* (RIE), are usually made available to users in microfiche, reproduced paper copy, and electronic media, and sold through the ERIC Document Reproduction Service (EDRS). Credit is given to the source of each document, and, if reproduction release is granted, one of the following notices is affixed to the document.

If permission is granted to reproduce and disseminate the identified document, please CHECK ONE of the following three options and sign at the bottom of the page.

The sample sticker shown below will be affixed to all Level 1 documents	The sample sticker shown below will be affixed to all Level 2A documents	The sample sticker shown below will be affixed to all Level 2B documents
PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL HAS BEEN GRANTED BY Sample TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC) 1	PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL IN MICROFICHE, AND IN ELECTRONIC MEDIA FOR ERIC COLLECTION SUBSCRIBERS ONLY, HAS BEEN GRANTED BY Sample TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC) 2A	PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL IN MICROFICHE ONLY HAS BEEN GRANTED BY Sample TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC) 2B
Level 1	Level 2A	Level 2B
☑	☐	☐
Check here for Level 1 release, permitting reproduction and dissemination in microfiche or other ERIC archival media (e.g., electronic) *and* paper copy.	Check here for Level 2A release, permitting reproduction and dissemination in microfiche and in electronic media for ERIC archival collection subscribers only	Check here for Level 2B release, permitting reproduction and dissemination in microfiche only

Documents will be processed as indicated provided reproduction quality permits.
If permission to reproduce is granted, but no box is checked, documents will be processed at Level 1.

I hereby grant to the Educational Resources Information Center (ERIC) nonexclusive permission to reproduce and disseminate this document as indicated above. Reproduction from the ERIC microfiche or electronic media by persons other than ERIC employees and its system contractors requires permission from the copyright holder. Exception is made for non-profit reproduction by libraries and other service agencies to satisfy information needs of educators in response to discrete inquiries.

Sign here,→ please

Signature: Robert P. Hunting	Printed Name/Position/Title: ROBERT P. HUNTING, PROFESSOR	
Organization/Address: MATH. DEPT., EAST CAROLINA UNIVERSITY GREENVILLE, NC 27858	Telephone: 252-328-1896	FAX: 252-328-6414
	E-Mail Address: huntingr@mail.ecu.edu	Date: 11/17/99

PS 027721

(over)

III. DOCUMENT AVAILABILITY INFORMATION (FROM NON-ERIC SOURCE):

If permission to reproduce is not granted to ERIC, *or*, if you wish ERIC to cite the availability of the document from another source, please provide the following information regarding the availability of the document. (ERIC will not announce a document unless it is publicly available, and a dependable source can be specified. Contributors should also be aware that ERIC selection criteria are significantly more stringent for documents that cannot be made available through EDRS.)

Publisher/Distributor:
Address:
Price:

IV. REFERRAL OF ERIC TO COPYRIGHT/REPRODUCTION RIGHTS HOLDER:

If the right to grant this reproduction release is held by someone other than the addressee, please provide the appropriate name and address:

Name:
Address:

V. WHERE TO SEND THIS FORM:

Send this form to the following ERIC Clearinghouse:	Karen E. Smith, Acquisitions Coordinator ERIC/EECE Children's Research Center University of Illinois 51 Gerty Dr. Champaign, Illinois, U.S.A. 61820-7469

However, if solicited by the ERIC Facility, or if making an unsolicited contribution to ERIC, return this form (and the document being contributed) to:

ERIC Processing and Reference Facility
1100 West Street, 2nd Floor
Laurel, Maryland 20707-3598

Telephone: 301-497-4080
Toll Free: 800-799-3742
FAX: 301-953-0263
e-mail: ericfac@inet.ed.gov
WWW: http://ericfac.piccard.csc.com