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Preschoolers' Counting and Sharing

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Both counting and sharing require action on discrete elements, entailing the logic of one-to-one correspondence. How counting and sharing relate to one another was the focus of an experiment conducted to examine strategies preschool children used to subdivide items. We designed tasks in which applications of counting skills, of visual cues such as subitizing, and of informal measurement skills were made more difficult. Children exhibited alternative strategies, suggesting use of a recipient as a mental cycle marker and an *adjacent* recipient strategy, with pauses between allocations suggesting a re-presentation of lots corresponding to the number of recipients. Results supported the view that dealing competence does not relate directly to counting skill.

Key Words: Early number learning; Preschool/primary; Early childhood; Whole numbers; Fractions; Clinical interviews

Mathematics, to the adult mind, is inseparably tied to notation and symbolism. As a result, children are generally given little credit for mathematical understanding before they begin school, although the process of constructing mathematical knowledge begins well before this time. Research by Rea and Reys (1970), Hendrickson (1979), Hughes (1986), Bergeron and Herscovics (1990), Young-Loveridge (1987), and Kamii (1985) has shown that young children have an impressive range of numerical skills at the onset of schooling. The *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 1989) stated that "children enter kindergarten with considerable mathematical experience, a partial understanding of many concepts, and some important skills, including counting" (p. 16).

Not only do young children come to school with a wealth of informal mathematical concepts and variable skills and abilities, there is evidence that their understanding of conceptual aspects of mathematics may not be manifest in their efforts to communicate using conventional symbolism. Carraher and Schliemann (1990) found that some children, with no previous school instruction on the numeration system, had a good understanding of conceptual aspects of the numeration system despite an inability to generate the number-name sequence systematically.

Most early mathematics learning is concerned with developing whole number knowledge, especially counting. Children from an early age spontaneously practice counting skills, including the conventional number-name sequence. The cognitive skills of both counting and sharing seem to develop during the early childhood years. Both skills require action on discrete elements, entailing the logic of one-to-one cor-

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respondence, as will be argued. Little is known about how the acquisition of these intellectual skills might relate to one another. Does knowledge of one skill serve in acquisition of the other? In particular, does the ability to count affect a child's ability to share discrete items, or is counting useful only for verifying the sizes of the shares created?

LITERATURE

The process by which children systematically allocate items resulting in equal shares has been reported in studies by Clements and Lean (1988), Davis and Pitkethly (1990), Hunting (1991), Hunting and Sharpley (1988a, 1988b), and Miller (1984). This process is also known as *distributive counting* (Miller, 1984), *splitting* (Confrey, 1995), or the *dealing procedure* (Davis & Pitkethly, 1990).

Although children have different understandings of what it means to share, evidence suggests that children can solve distribution and sharing problems before the related mathematical operations and procedures are taught in school (Hunting & Sharpley, 1988a, 1988b; Sebold, 1946). In his study of children's measurement procedures and the development of quantitative concepts, Miller (1984) suggested that children are attuned from an early age to equality of division. Frydman and Bryant (1988) proposed that high proficiency demonstrated by 3-year-old children in equally distributing items may be a repeated drill or repetitive action learned from others. Alternatively, Riess (1955) suggested that life experiences, such as the apportionment of food, may facilitate the early development of what it means to share. In a study of home influences on 3-year-olds' prefraction knowledge, including sharing, Hunting (1991) found the strongest positive factors were siblings, especially for those children in families with older siblings, and gender. Females generally performed better than males. Children in full-time daycare, those who played with a larger number of children, as well as those who engaged in tasks such as table setting were considered likely to perform better on sharing tasks, but evidence to support these conjectures was not found.

Davis and Pitkethly (1990) argued that although preschool children will use a dealing strategy in a structured situation, they are unaware that dealing in itself is adequate to establish fair shares. Frydman and Bryant (1988) went further to claim, "If children have a full and explicit understanding of the quantitative significance of sharing, they should be able to infer the number of items in one shared set when they know the number in the other" (p. 325). A distinction between social sharing and mathematical sharing was made by Desforges and Desforges (1980). If children were to view the distribution of such items as sweets as a social action, the mathematical importance of this act might be lost, eliminating the need to use a dealing strategy to establish fair shares.

Although systematic sharing has been observed in structured interview situations in which discrete items were to be distributed, Davis and Hunting (1991) found a total absence of sharing in an informal situation without an adult present. Other factors that may influence children to deal in a structured interview, such as child-and-

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adult interaction, but ignore this procedure in a more informal situation have not as yet been fully investigated.

Counting

Theories explaining how children learn to count differ with respect to development of counting processes. Fuson and Hall (1983) and Fuson (1988) have emphasized the role of language patterns as children acquire the conventional number-word sequence. A counting model in which the young child's developing counting skills are principle-driven has been proposed by Gelman (e.g., Gelman & Gallistel, 1978; Gelman & Meck, 1983, 1986, 1992). Gelman and Gallistel (1978) proposed five counting principles that they believed underpinned accurate counting: a one-to-one principle, a stable-order principle, a cardinal principle, an abstraction principle, and an order-irrelevance principle. Studies designed to clarify how the order in which items of a collection are counted affects the count outcome have been conducted by Baroody (1984, 1993), Gelman, Meck, and Merkin (1986), and Gelman and Greeno (1989). The production of qualitatively different unit items in children's solutions to numerical tasks has been proposed by Steffe, von Glasersfeld, Richards, and Cobb (1983) and Steffe and Cobb (1988). Fuson's (1988) analyses of counting behavior and the principles set out by Gelman and Gallistel (1978) are based on skills the child is able to demonstrate. Gelman and Gallistel were concerned that the counting abilities of many children were being underestimated because assessment of these abilities had for too long been based on what children could not do rather than on the skills they exhibited.

Steffe et al. (1983) acknowledged that initially children must acquire the number-word sequence. However, a concurrent cognitive hurdle is isolation of discrete items to which the individual number words would be coordinated. A further hurdle is the ability to coordinate the utterance of each number word with one item among those that are to be counted. Other investigators (e.g., Gelman & Gallistel, 1978; Ginsburg, 1977; Herscovics, Bergeron, & Bergeron, 1987) have conducted investigations of the development of one-to-one correspondence and its relationship to the oral counting sequence. Although Steffe and his coworkers do not ignore the significance of one-to-one correspondence and its contribution to the initial development of counting, they have attempted to demonstrate that the development of unit items to be counted is of crucial importance.

Steffe and Cobb (1988) proposed that advances in counting competence are made through increasing independence of perceptual items and sensory cues to a stage in which abstract unit items can be created and counted. Children can be classified at different levels of counting development according to the type of unit item they produce when counting:

1. Counters of perceptual unit items—require perceptual items to establish units that can be counted. With the presence of direct sensory material, children are able to solve simple additive problems. Without a collection of items, they are unable to carry out the activity.

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2. Counters of figural unit items—are able to use figural or pictorial representations for coordination with the number-word sequence. That is, in a simple additive task in which some items are hidden, the child makes a representation of the missing items. Important in this stage is the development of visual patterns and finger patterns.

3. Counters of motor unit items—can take a motor action (e.g., a rhythmic hand movement, pointing act, or act of putting up a finger) to stand for a countable unit item that has been hidden. A substitution is made for the previously counted perceptual or figural item.

4. Counters of verbal unit items—are able to take their own utterances of the number names as countable unit items. Children are still not numerical, however, because numerosity is not embedded in the meaning of the number name when it is uttered. The transition from being a counter of perceptual unit items to a counter of verbal unit items involves the internalization of sensory-motor activity.

5. Counters of abstract unit items—are able to take a number as implying a count that has already been performed and are therefore likely to count on in simple additive tasks. That is, the child can take a number word, say *eight*, as implying that the count "1, 2, 3, ... 8" has already occurred and will continue counting on from that number.

Steffe and Cobb (1988) proposed that "counting types indicate what children's initial, informal numerical knowledge might be like, and reflect our contention that children see numerical situations in a variety of qualitatively different ways" (p. 1).

A link between counting and sharing. Studies reviewed suggest that children can solve distribution and sharing problems before the related mathematical operations and procedures are taught in school (Hunting & Sharpley, 1988a, 1988b; Sebold, 1946). Frydman and Bryant (1988) reported other convincing empirical evidence that children as young as 3 years can manage quite well in informal tasks in which they have to share discontinuous quantities among two or more people. These children do so on a one-for-him, one-for-her basis, which, on the surface at least, looks very like a one-to-one correspondence (Desforges & Desforges, 1980; Miller, 1984). According to Beckwith and Restle (1966) and Gelman (1972), counting involves pairing items from a physical set with a set of psychological markers such that a one-to-one correspondence is preserved between items and markers. Steffe et al. (1983) understood counting as the coordination of a counting word and a countable unit item. A logical analysis of activities of both counting and sharing indicates that one-to-one matching is a common feature.

At least part of the cognitive task of counting is needed to share items equally, namely, application of a one-to-one matching. On theoretical grounds, then, it seemed reasonable to hypothesize that children who were better at sharing would be more competent counters. This hypothesis was only partially supported in a preliminary study with preschool children (Pepper, 1991). No relationship was found in that study between counting and solving a sharing task in which 12 crackers were to be distributed equally between two dolls. However, when a third doll was introduced and

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redistribution was required, the relationship between counting and sharing was significant.

The majority of children (93%) were successful in solving the first sharing task, in which 12 crackers were to be distributed equally between two dolls. The method employed by most children was that of a systematic dealing strategy in which the crackers were distributed on a one-to-one basis, in a cyclic pattern until all the crackers had been given to the dolls. Most children stacked the crackers in front of the dolls. Variations on this method were also observed. Approximately half the children who gave the dolls even shares gave no indication that they knew equality had been achieved. One possible explanation for lack of justification was that children's language skills were not sufficiently developed to allow them to explain the situation. Other possible methods used for equalizing shares might have included visual comparisons of the height of stacks or the length of rows. It was also possible that children were able to subitize (Kaufman, Lord, Reese, & Volkmann, 1949) quantities while the items were being distributed.

The second sharing task elicited a much greater variety of response. Counting was more widely used not only for the verification of share sizes but also during the process of sharing. A major issue raised was how those children classified as poor counters were able to equally divide groups of discrete items and to be confident about the result. Was achievement of equality possible through the use of prenumerical skills such as subitizing, or were informal measurement skills such as visual comparison important, or were possibly a combination of skills used?

METHOD

Twenty-five preschool children aged from 4 years 10 months to 6 years 2 months (average age, 5 years 4 months) participated in two interviews administered on different occasions. One interview consisted of a set of counting tasks; the other a set of sharing tasks. Each interview took 15–20 minutes on average, depending on the child's responses. All interviews were videotaped for later analysis.

Counting Tasks

The counting tasks were designed to investigate the level of development that children had reached according to the Steffe et al. (1983, 1988) theory of counting types. From the counting interview the children were classified into three groups: *poor*, *developing*, and *good* counters:

1. *Poor counters* were able to solve only problems for which perceptual material was available. These children, according to the theory of Steffe et al., are counters of perceptual unit items.

2. *Developing counters* were children who seemed to produce visualized images of the hidden items but seemed unable to count them to solve the problem. These children are said to be counters of figural unit items and are considered to be in a transitional stage between being counters of perceptual unit items and motor unit items.

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3. *Good counters* were able to count motor actions or verbal utterances or were able to operate without depending on sensory-motor material. These children are said to create motor, verbal, or abstract unit items.

A farm scenario was used for the counting interview. The scenario involved seven tasks involving small toy animals and people and structures made with Lego bricks to represent sheds and houses, arranged on a board measuring 2 ft by 3 ft (see Figure 1). Initial tasks provided perceptual information for the child. Subsequent tasks became increasingly difficult as perceptual information was restricted. The tasks were structurally similar to part-part-whole situations used to investigate children's knowledge of arithmetic operations (Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993). However, the focus of analysis was on the level of counting competence. The child was given time to become familiar with the setting and to name the animals on the farm before being asked to solve any problems. A description of the problems follows.

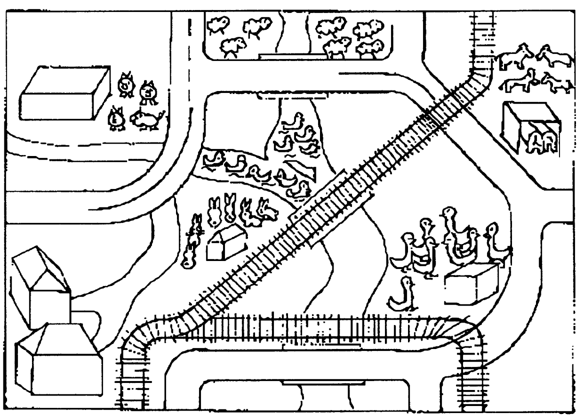

Figure 1. The farmyard-counting-tasks board.

Problem 1: The child was shown two groups of white sheep, five in one paddock and four in the other. The child was asked to find the total number of sheep after being told the number in each group.

Problem 2: The child's attention was drawn to two groups of ducks on a pond, five red and four yellow. The child was again told the number in each group and was asked to find the total number of ducks.

Problem 3: The child was shown eight cows in a paddock, six visible and two hidden inside a shed. The child was shown the cows in the shed, which was quickly removed and then replaced. The total number of cows was to be found. The number of cows outside the shed was not mentioned by the interviewer.

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Problem 4: There were nine rabbits in a yard, three hidden in a hutch and six visible. The child was told that three shy rabbits were hiding in the hutch so they could not be seen. The child was then asked to find the number of rabbits there were altogether, including the three rabbits in the hutch.

Problem 5: This problem was similar to the previous problem. Seven pigs were hidden in a shed and four were outside the shed. The children were again not shown the pigs in the shed, but were told the number that were hidden and were asked to find the number of pigs on the farm.

Problem 6: Four geese were placed inside a shed and 8 geese outside the shed. The child was told that there were 8 geese outside the shed and that there were 12 geese altogether. The child was asked to find the number of geese hiding in the shed.

Problem 7: The child was shown two houses and was told that there were five people in one house and six in the other. No people were shown, but the child was asked to find the total number of people.

Also included in the counting interview were verbal tasks to assess the stability and understanding of the verbal counting sequence between 1 and 20. This information, although not of direct interest, was included to provide useful background information to solutions of the farmyard problems. For example, unstable or incorrect verbal number sequence performance might help explain an incorrect answer to a counting problem.

Subitizing tasks were also included in the counting interview to investigate children's abilities to identify quickly, without counting, the number of items in a small group. For the first part of this task, the children were required to match an array of dots on a flashcard with one of four arrays on a display board (see Figure 2). There were four flashcards having different arrays. The order of presentation of flashcards did not correspond to the order displayed on the board. For the first part of the task, the dots on the flashcards were in the same configuration as the dots on the board. For the second part of the task, the children were again quickly shown flashcards with either 2, 3, 4, or 5 dots. In this case, the numbers of dots were the same but the configurations were different (see Figure 2). The children were again requested to identify the display with the same number of dots as they had seen on the card. Display time for the flashcards was controlled by the interviewer and was approximately 0.5 second.

Sharing Tasks

The following sharing tasks were designed to limit the type of information available, thus helping to clarify how children were able to equally subdivide groups of discrete items. A description and rationale for each task follows.

Task 1. The children were shown two dolls and were told that, after playing, the dolls were very hungry. The children were then given 12 crackers on a plate and were asked to share these between the dolls so that each doll received an equal share.

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The children were asked to determine whether equal shares had been given to each doll after all the crackers had been distributed. Dolls used were approximately 100 mm in height; crackers were flat and circular with diameter approximately 35 mm.

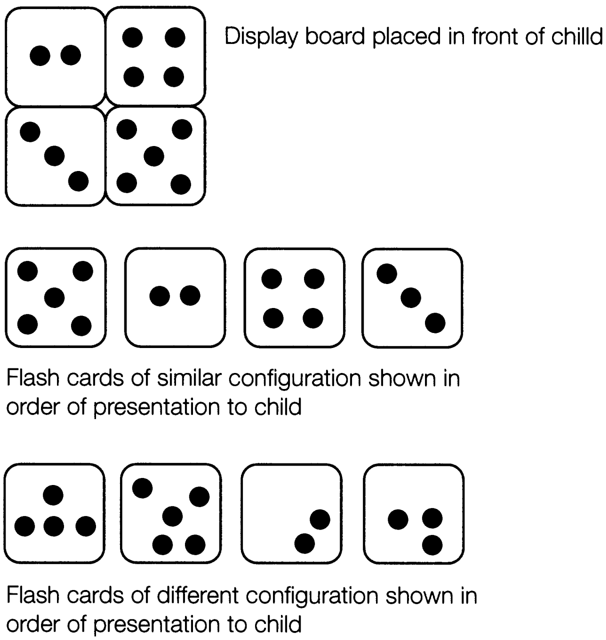

Figure 2. Display cards used during subitizing experiment.

Rationale. The crackers could be stacked so visual comparison was possible during both the dealing procedure and the verification of equality of shares at the completion of dealing. While the crackers are being distributed, it is possible that subitizing occurs to establish equality after each cycle of the distribution. The likelihood of this eventuality decreases as the size of each share increases beyond five. This task is identical to the first sharing task in an earlier study (Pepper, 1991), thus allowing for comparison with the first group of children to establish whether similar results were achieved.

Task 2. A third doll was introduced, with all three dolls becoming hungry after finishing some jobs. The children were then given 21 "Tiny Teddy" cookies that they were asked to share evenly among the three dolls. (Tiny Teddy cookies are approximately 25 mm long, 15 mm wide, and 5 mm thick and are molded in the shape of a small bear.) Justification for even shares was again requested from the children. The dolls were arranged in a line in front of the child with the third doll to the child's right.

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Rationale. Tiny Teddy cookies cannot be stacked, so visual height comparisons are eliminated. It is still possible to make lines of cookies and compare the line lengths. It was anticipated that this constraint would mean a change of strategy from the first task, in which most children were expected to stack the crackers. This expectation was based on previous results (Pepper, 1991). Increasing the number of items to be distributed, resulting in bigger shares being received by each doll, greatly reduces the possibility of accurately subitizing the final shares to establish equality. Subitizing during the process of distribution is still possible.

Task 3. The children were told that the dolls had done such a good job that they could have some pocket money. Identical metal money boxes, of dimensions 80 mm by 55 mm by 90 mm, were placed in front of each doll, and a stack of 15 twenty-cent coins was placed on the table. The children were asked to distribute the money into the money boxes so each doll got an even share. At the completion of distribution, the children were asked, "Has each doll got an even share? How can you tell?"

Rationale. Placement of the coins into the money boxes while they are being distributed precludes the possibility of visual comparison of the shares. Subitizing of shares is possible neither during nor at the completion of distribution because the coins cannot be seen. The coins are not visible after being placed in the money boxes, so strategies of visual comparison and subitizing are unlikely to be used. Some other process would be necessary to regulate a procedure for giving equal shares. Internal monitoring of units as they are distributed, the use of some kind of signpost at the commencement of each cycle, or perhaps a combination of both skills would be possible. It is expected that even if children are confident about equal shares being given, they will be unable to state the numerical size of each share.

The sharing tasks required the children first to distribute 12 crackers between two dolls. A third doll was then introduced, requiring the crackers to be redistributed among the three dolls. The children were again classified into three groups according to the following criteria:

Good sharers created equal shares and either used a systematic method or completed sharing in three or fewer moves.

Intermediate sharers created equal shares and either displayed some systematic method or completed sharing in four to seven moves.

Poor sharers created unequal shares or used more than seven moves.

RESULTS

Results of subitizing tasks will be discussed with particular attention to how they compared with children's counting competence. Results of the analyses of children's responses to the counting and sharing tasks will be presented separately, followed by an analysis of the relationship between counting and sharing competence.

Subitizing

The first task required the children to point to a display that had the same num-

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ber of dots as on the given flashcard. For this task the spatial arrangement of the dots on each flashcard corresponded to the spatial arrangement of dots on the display (see Figure 2). Twenty-three children (92%) interviewed gave correct responses to the presentations of all four cards. One child gave no response. One other child made two errors. She pointed to the display showing four dots in response to the flashcard with five dots and, when shown four dots, initially pointed to the display with three dots but then changed her mind and pointed to the display with five dots.

For the second task, for which the configuration of dots on the flashcards differed from the display in front of the child, the number of correct responses decreased dramatically, especially for the recognition of four and five dots (see Table 1). Three children (12%) incorrectly matched each of the four flashcards. Two children responded correctly to all four flashcards, and again one child gave no response for any of the cards.

Table 1
Observed Frequencies for Correct Responses to Subitizing Cards (Similar Patterns)

	Number of correct responses			
Type of presentation	2 dots	3 dots	4 dots	5 dots
Similar configuration	24 (96%)	24 (96%)	23 (92%)	23 (92%)
Different configuration	21 (84%)	13 (52%)	2 (8%)	11 (44%)

Table 2 shows the numbers of children classified according to the counting tasks and the second subitizing task. Incorrect responses were distributed across all categories of counting competence. Of interest were the responses of two children, both classified as poor counters. These children correctly matched the number of dots on each flashcard with the number on the display, even though the configuration of dots on the display differed from that on the flashcards. It was also of interest that no children classified as good counters were able to correctly identify all four flashcards. A Spearman rank-order procedure was used to test the strength of the relationship between counting and subitizing performance for this task. No relationship was found between children's counting ability and their performance on the subitizing tasks.

Table 2
Observed Frequencies for Counting and Subitizing Performance (Different Patterns)

	Subitizing			
Counting	All correct	1 or 2 incorrect	3 or 4 incorrect	Total
Good	0	3	1	4
Developing	0	5	1	6
Poor	2	9	3	14
Total	2	17	5	24

Spearman's r_s (corrected for ties) = .06; p = .78.

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Farm Tasks

On the farm tasks, 4 children (16%) were classified as good counters, 6 (24%) were classified as developing counters, and 15 (60%) were classified as poor counters. The percentage of children classified into each category was similar to that found in the earlier study (Pepper, 1991). Children classified as poor counters were either unable to solve any of the farm problems or able to solve only problems for which perceptual material was available. The rabbit problem, which was added for this experiment, helped to clarify which children were counters of perceptual items. Unlike the cow problem, for which the children were quickly shown the hidden cows, no perceptual information was given for the rabbit problem. Many of the children used pointing when counting to solve the problems. Some children were unable to solve any problems. For both the cow and the duck problems, the children were able to count the smaller groups but were unable to find the total number. Some children incorrectly counted the total number. If the children incorrectly counted when solving the cow and duck problems, then they were also likely to count incorrectly when solving subsequent problems. The children in this category had no strategy for attempting the pig problem, so the interview was terminated at this stage.

Children classified as developing counters were able to solve problems for which perceptual information was available. They correctly counted all the white sheep and then counted the red and yellow ducks on the pond. They had no difficulty dealing with the two cows in the shed when finding the total number of cows. Four of the six children in this category successfully solved the rabbit problem, usually by first counting the visible rabbits and then pointing three times over the shed as they counted. The two children in this category who did not solve the rabbit problem both pointed over the shed as they counted but only pointed twice, reporting a total of eight rabbits. All the children in this category continued to use this method of counting the visible items first when solving the pig problem. All children were unsuccessful since they were unable to keep track of the number of counts they had performed. A total of eight or nine pigs was usually reported.

Children classified as good counters were not able to solve all the farm tasks, but they demonstrated strategies that, if developed further, are more likely to lead to successful solutions of the problems. These children had no difficulty solving problems for which perceptual information was available. Unlike the developing counters, these children counted the hidden items first, using a counting-on strategy, when solving the rabbit and pig problems, thus enabling them to keep an accurate record of their counting. No good counter was able to solve the geese problem, which required finding the number in the shed given the total number. Most children preferred to look through the shed door and guess. Only one child found the total number of people in the two houses; she was unable to describe her own strategy.

Sharing Task 1

For the first sharing task, which involved the sharing of 12 crackers evenly between two dolls, 23 (92%) of the children were successful in evenly distributing the crack-

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ers. The predominant method of distribution was a systematic dealing strategy. This strategy generally involved six "one cracker for one doll" cycles. Three children were observed to distribute the crackers using two hands simultaneously. Five children used counting during the process of distributing the crackers. These five children were all classified as good or developing counters. Of the children who distributed the crackers systematically, only one child was observed to alternate the beginning point of a cycle. That is, instead of consistently beginning a distribution cycle with Doll 1, the child alternated between Doll 1 and Doll 2 when beginning a cycle. This child was classified as a poor counter on the farm tasks. The majority of children made stacks of crackers in front of the dolls.

Both children who were unsuccessful at solving this task started by systematically sharing the crackers between the two dolls. One child confused the two piles he was making as he distributed the crackers, creating confusion as to which doll should receive the next cracker. The second child, after systematically distributing four crackers, divided the remaining pile of crackers into two unequal groups and gave one group to each doll.

Sharing Task 2

For this task, in which the children were asked to share 21 Tiny Teddy cookies evenly among three dolls, a wider variety of solution strategies was observed. Fifteen (60%) of the children successfully distributed the cookies evenly among the three dolls. The use of a systematic dealing procedure was evident in many of the children's successful solutions, although counting appeared to play a more significant role in the children's strategies for solving the problem. The following describes the successful solutions that were used by the children to solve the problem and the number of children using each strategy.

1. Give one cookie to Doll 1, one cookie to Doll 2, and one to Doll 3; repeat this cycle seven times (six children). Not all children commenced each cycle with the same doll. One child placed the cookies into lines during the distribution process. The other children stacked the cookies.

2. Give one cookie to each doll; repeat four times. The last cycle was modified so that two cookies were given to each doll during this cycle. The child who solved the problem this way was unsure if the dolls had been given equal shares (one child).

3. Give one cookie to each doll, repeat three times, then give two cookies to each doll, finishing with a one-cookie-to-one-doll cycle (one child).

4. Give one cookie to each doll, repeat two times, commencing with Doll 3 on each cycle. The fourth cycle commenced with Doll 2, and only Dolls 1 and 2 were given cookies. For the fifth cycle, each doll was given three cookies, resulting in one cookie remaining. This cookie was given to Doll 1. The child then spread out each doll's cookies in front of that doll, appearing to make the same pattern with each group. One cookie was then taken from Doll 1 and given to Doll 3 so that equal shares resulted (one child).

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5. The child started distributing the cookies systematically with four one-cookie-for-each-doll cycles. Each cycle was commenced at the same doll. The child was then distracted and randomly distributed the remaining cookies. When all the cookies were distributed, the child used counting and possible similar pattern placement to make the shares equal (one child).

6. Give three cookies to each doll, followed by four one-cookie-for-each-doll cycles. Each cycle was commenced with a different doll (one child).

7. Give four cookies to each doll, with counting being used to verify the size of each pile after completion of the cycle. The second cycle was a two-cookies-for-each-doll cycle followed by a one-cookie-for-each-doll cycle. All cycles were commenced with the same doll, and counting was used at the conclusion to verify the size of each share (one child).

8. Six cookies were counted out and given to Doll 1, then six more for Dolls 2 and 3. There was then a one-cookie-for-each-doll cycle. The size of each share was verified through counting (one child).

9. Six cookies were counted out for each of Dolls 1 and 2. One more cookie was given to Doll 1 with the utterance, "That makes seven," and repeated for Doll 2. Five cookies were then given to Doll 3, shares being confirmed through counting. The remaining cookies were then given to Doll 3 as the child counted, "six, seven." Equality was justified by saying, "They've all got seven" (one child).

10. Six cookies were counted out to each of Dolls 1 and 2, and seven cookies for Doll 3. Each share was then counted, with one cookie being removed from Doll 3. The remaining three cookies were then given one to each doll (one child).

Sharing Task 3

Although it was originally thought that Task 3 would be the hardest problem for the children to solve, 16 children (64%) were successful in distributing the 15 coins equally among the three money boxes. The 16 children who were successful in solving the task all used a systematic procedure for distributing the coins. The methods were as follows:

1. One-coin-to-one-money-box cycle repeated five times. Fourteen children used this method, with 11 always starting a cycle at the same money box, whereas 3 varied the commencement point of each cycle. While the children were distributing the coins, some were seen to pause with a hand over a money box, as if replaying in their heads the actions already performed. In most cases the initial response was changed and equality maintained through the cycle. One child was observed to pick up three coins from the pile at the beginning of each cycle.

2. One child counted five coins into each money box, justifying equality by saying, "They've all got five."

3. One child began with a cycle of three coins for each money box followed by two cycles of one coin for each box.

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Of the children who were unsuccessful at solving the task, most demonstrated some evidence of systematic sharing at the beginning of distribution but were unable to sustain this method for the total distribution of coins.

Observed Frequencies for Counting and Sharing

By classifying the children according to their performance on both the counting and sharing tasks, we developed cross-tabulations to establish what relationship, if any, existed between children's counting competence and their abilities to share (see Tables 3, 4, and 5). Sharing responses were categorized according to whether systematic dealing had been used, the distribution was random, or equality of the shares had been achieved.

Table 3
Observed Frequencies for Counting and Performance on the First Sharing Task

	Sharing			
Counting	Unsystematic and unequal shares	Unsystematic and equal shares	Systematic and equal shares	Total
Good	0	0	4	4
Developing	0	1	5	6
Poor	2	0	13	15
Total	2	1	22	25

Spearman's r_S (corrected for ties) = .1; p = .61.

Table 4
Observed Frequencies for Counting and Performance on the Second Sharing Task

	Sharing			
Counting	Unsystematic and unequal shares	Unsystematic and equal shares	Systematic and equal shares	Total
Good	0	1	3	4
Developing	3	1	2	6
Poor	7	1	7	15
Total	10	3	12	25

Spearman's r_S (corrected for ties) = .17; p = .40.

Table 5
Observed Frequencies for Counting and Sharing Performance on the Third Sharing Task

	Sharing			
Counting	Unsystematic and unequal shares	Unsystematic and equal shares	Systematic and equal shares	Total
Good	1	0	3	4
Developing	2	0	4	6
Poor	6	0	9	15
Total	9	0	16	25

Spearman's r_S (corrected for ties) = .11; p = .58.

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Tables 3, 4, and 5 show the observed numbers of children classified according to the counting and sharing tasks. A Spearman rank-order procedure was used to test the relationship between counting and sharing performance for each of the sharing tasks. For each sharing task, a small positive but statistically nonsignificant relationship was found between counting competence and the ability to share. A large number of poor counters were able to solve each sharing task. For the first sharing task, 87% of children classified as poor counters successfully created equal shares. The number of poor counters who solved the second task decreased to 53%, but the success rate for poor counters rose to 60% for the third sharing task. Of the children classified as good counters, 75% were able to solve all sharing tasks. All developing counters were able to solve the first sharing task. For the second task, 50% of these children solved the task, whereas, like the poor counters, more (66%) were successful at solving the third sharing task.

DISCUSSION

For each of the sharing tasks, there was no significant relationship between counting and sharing competence. Sharing Task 1, in which 12 crackers were shared between two dolls, produced similar results to that observed in the earlier study (Pepper, 1991). The predominant method for solving this task was a systematic dealing procedure. Children who were unsuccessful at solving this task appeared to become distracted by the presence of the video monitor, resulting in unequal shares being given. One notable behavior was that three children used counting as an integral part of the distribution process. These children used a systematic dealing procedure to distribute the crackers, but counting was used during the process to verify the size of each doll's share. All three children were classified as either good or developing counters on the basis of their performance on the farm tasks. For these children counting may have been triggered by the act of sharing. It was unlikely that children were influenced to count because of the counting interview, which had been conducted on a previous day.

Sharing Task 2, in which 21 Tiny Teddy cookies were shared among three dolls, produced a larger variety of solutions from the children. Many children still used a systematic dealing procedure to distribute the Tiny Teddy cookies, although it became more common to observe the use of many-to-one distributions. For Task 1, the crackers were generally distributed one cracker to one doll. For Task 2, the Tiny Teddy cookies were more often distributed with two or more cookies being given to each doll during a cycle. This many-to-one distribution may have been a consequence of the size of the cookies. The children may have related the Tiny Teddy task to similar contexts in the home. If cookies are smaller, one usually is allowed to eat more of them. The larger number of cookies to be distributed may have forced some children to look for a quicker method of distribution. Rehearsal from the first task may also have had an influence on the method used. Children who were confident about the result they achieved by distributing the crackers on a one-for-one basis during Task 1 may have taken advantage of the opportunity to refine the process.

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A consequence of systematic dealing is that maintenance of equality through a cycle results in equal shares even if the size of shares differs from one cycle to the next.

Task 2 was designed to eliminate the possibility of height comparison to see if another method would be developed as a check for equality between shares. It appeared that some children achieved equality by placing cookies in the same pattern in front of the dolls. Other children made rows of cookies in front of each doll and compared the lengths of the rows, not always successfully, depending on how straight the rows had been made. The three children who in Task 1 used counting to verify share sizes during the distribution repeated this procedure in Task 2. Two other children used counting during this task when distributing the cookies, one child being classified as a good counter, the other as a developing counter.

Task 3, in which 15 coins were to be distributed into three money boxes, was considered to be the most conceptually difficult. This task was designed to restrict use of strategies the children may have used when solving the previous tasks. After the coins were deposited into the money boxes, the children were no longer able to manipulate the coins or visually check the size of shares. Because the children were prevented from seeing and being able to manipulate the shares while they were distributed, we felt that most children would be unlikely to achieve equality among shares. However, we found that children did demonstrate alternative strategies to solve the problem. A common strategy used by many children to solve this task was use of a systematic dealing procedure, with each distribution cycle beginning at the same money box. That is, if the first cycle began with Doll 3, then all other cycles commenced with this doll. One child who distributed the coins in this way was seen to pick up three coins from the pile before the commencement of each cycle. This child, and possibly others, may have been using a unit of 3 as a way of maintaining equality. It is also possible that children mentally marked a particular doll as a signpost to indicate where a new cycle would commence. Such a strategy is plausible considering that the children who commenced a cycle with the same doll never chose the middle doll as a starting point. Another feature of some children's distributions was an "adjacent box" strategy whereby they would deposit the next coin into the box adjacent to the box into which the previous coin had been deposited. Before a coin was deposited, the child would pause as if replaying actions already performed during the cycle. At this time the correct decision was made as to where that coin should be deposited. It is possible that these children were keeping a mental record of their actions. Were children who varied the starting point of each cycle using a variety of strategies? It is possible that these children were choosing a doll as a signpost at the commencement of each cycle and then either using units of 3 to complete a cycle or keeping mental records of the actions performed during a cycle. In this situation, a child would need to re-present lots of 3 in coordination with the cognitive skill of one-to-one matching, a skill that develops with early counting skills. Hence, ability to equally distribute groups of items would seem not to depend on advanced counting skills. Counting is a way to verify the results of the action scheme of sharing. A method of justification sometimes used instead of counting was to verbally report a replay of the sequence of actions performed.

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For example, Emily (5 years 2 months) explained, "One got one, another got another, and another got another." David (5 years 4 months) explained by saying, "I keep giving them the same much."

Matthew (6 years 2 months), who had used counting to solve the first two sharing tasks, immediately recognized the difficulty of depositing coins into money boxes where the coins could not be seen. He placed groups of coins in front of each money box, counting the number in each group. Because of a counting error, two coins remained after the others had been deposited. The child had no way of deciding into which boxes the coins should be placed. He tried unsuccessfully to remove coins from boxes, then peered down the slots in the money box. For this child, reliance on counting skills may have interfered with engagement of other strategies.

Of the children unsuccessful in solving Task 3, most demonstrated some evidence of systematic distribution. They would begin sharing systematically but would be unable to maintain order for the total distribution of the coins. These children may have been distracted during the task or may have failed to establish when one cycle finished and another began. These children were unable to tell whether equal shares had been given, the common response being one of uncertainty.

It was hoped that insight could be gained into children's partitioning and sharing strategies through observation of their behavior in solving the subitizing tasks. It was felt that children who correctly responded to all flashcards might be able to use this skill when solving the sharing tasks. However, no conclusive evidence could be found to support this conjecture. Sarah (5 years 5 months) and Kristy (5 years 4 months) were both classified as poor counters. Both correctly identified the subitizing cards when the configurations of dots were the same. Both were unsuccessful when the configuration of dots on the flashcards differed from those on the display in front of them. Yet both girls successfully solved all three sharing tasks through the use of systematic dealing procedures. This result suggests that methods other than subitizing or perhaps counting were being used to create equal shares. Travis (5 years 5 months) was also classified as a poor counter. He was able to correctly identify all flashcards in the subitizing tasks. He successfully solved the first two sharing tasks but was unable to solve the third task, for which the coins were hidden after distribution. This result suggests that subitizing may have been a strategy used by this child in creating equal shares. Results from this experiment are inconclusive as to what role subitizing skills play in children's partitioning and sharing abilities.

Suggestions for Teaching

This research suggests that dealing competence does not relate directly to counting skills. The children studied demonstrated systematic dealing procedures, involving no apparent use of counting or measuring skills, that resulted in groups of discrete items being partitioned equally. Children classified as good sharers exhibited different degrees of counting competence. Teachers should be able to involve young children in problems of sharing even if those children have not yet become rational counters. Indeed, dealing tasks may assist children's developing counting skills through opportunities to check or verify the sizes of shares. The type of task given to the chil-

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dren will determine the extent to which counting may be needed. Tasks such as sharing 12 crackers between two dolls can be successfully completed without the use of counting skills, whereas tasks that involve larger numbers of items may encourage the use of counting. Children using other means of comparing quantities, such as subitizing, may have to rely on different methods to verify shares when the number of items increases. Engagement in tasks that entail shares of more than six items could encourage children to rely more on explicit quantitative methods, such as counting or visual comparison of space occupied. Task settings that prevent children from scanning relative sizes of shares could also be presented. Such settings could be achieved, for example, by placing items, as they are shared, into small containers such as money boxes, so that perceptual cues are eliminated. Children encouraged to become less reliant on perceptual information and more reliant on internal methods will develop more sophisticated schemes that, in turn, will allow them to extend their knowledge to deal with new and more challenging situations. For problems with small numbers, such as eight coins to be distributed equally into two opaque boxes, children could be allowed to see the coins in one box but only be told the number of coins in the second box. This practice would encourage counters of perceptual items to point or make verbal counts for the hidden items if they are to find the total number of coins. In this situation, a child may realize that the verbal utterance "four" can stand for four items that no longer need to be counted. Such a teaching strategy could be gradually extended to include problems with larger numbers. Through group work, teachers could have children discuss ways in which different-sized groups might be shared, with discussion centered on advantages and disadvantages of using systematic and nonsystematic sharing procedures. Fraction names could be informally introduced, beginning with one half. The notion of division would also be dealt with, allowing for conceptual development, again before formal written notation is introduced.

Many children of preschool age have shown they have sufficiently well developed counting skills to be able to solve simple addition and subtraction problems that are set in relevant contexts, provided that perceptual information is available. Some children of this age may be ready to attempt relatively complex tasks in which small numbers of discrete items are hidden from view. The teaching of addition and subtraction is usually formally introduced to children during the second half of their first year at school. If teachers assessed children early in the school year, using clinical tasks such as those used in this study, it is likely that they would identify children who are capable of attempting more challenging work. Teachers could then introduce these concepts earlier by presenting problems such as those in the farm setting. In this way the children could develop conceptual understanding of addition and subtraction before formal written notation is introduced. Counting skills would also be learned and practiced while children actively engage in solving these types of problems.

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